PROCTOR'S STATEMENT

This is to certify that ______________________________________ wrote an examination in the course _____**Math 208**______ under my personal supervision and received no outside aid from any source whatsoever. The student was verified through a picture ID prior to taking the examination. The completed examination is being sent to the Online and Distance Education Office by me.

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Signature of Examination Proctor

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Position

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Date

Math 208 Correspondence Lesson 14: Exam 2 Name: ____________________

TIME LIMIT: 2 hours; no books, no notes, no calculators, etc

I: ($2\frac{1}{2}$ points each) True or False (circle your answer):

1) True False: If $\lfloor x \rfloor = \lceil x \rceil$, then x is an integer.

2) True False: $\sum_{k=1}^{100} k^2 = \sum_{k=0}^{99} (k+1)^2$.

3) True False: If $a_0 = 3$, and for $n \geq 1$, $a_n = 2 - a_{n-1}$, then $a_{100} = -1$.

4) True False: The set described recursively by (a) $1 \in S$, and (b) if $k \in S$, then $2k \in S$ is the set of even positive integers.

5) True False: It may be possible to prove a theorem using the second form of induction when the first form of induction will not work.

6) True False: The well ordering property of the natural numbers is used to prove that induction is valid method of proof.

7) True False: The worst case scenario function for an algorithm gives approximately the maximum number of steps the algorithm will use for all problems of a given size.

8) True False: $2n + 3n^2$ is $O(n^2)$.

9) True False: $\forall n \in Z, -1 | n$.

10) True False: 1573 is a prime.

11) True False: The smallest positive integer that can be written as a linear combination of 231 and 195 is 1.

12) True False: If a, b, c are positive integers, and $a < b$, then $\gcd(a, c) < \gcd(b, c)$.

II: (5 points each) Multiple Choice (circle your answer):

1) The sum of the first 200 terms of the arithmetic sequence with initial term 2 and common difference 3 is

   (a) 599
   (b) 601
   (c) 604
   (d) 60100
   (e) 60400

2) According to the laws of exponents, $a^b \cdot a^c$ equals

   (a) a^{bc}
   (b) $(a^2)^{b+c}$
   (c) $(a+a)^{bc}$
   (d) a^{b+c}
   (e) None of the above.

3) A geometric sequence begins $a_0 = 2$, $a_1 = 6$. The value of a_4 is

   (a) 18
   (b) 36
   (c) 54
   (d) 162
   (e) There is not enough information to determine a_4.

4) A set S of strings over the alphabet $\Sigma = \{a, b, c\}$ is described recursively be (1) $a \in S$, and (2) if $x \in S$, then $bxc \in S$. Circle all the true statements in the list below.

   (a) Every string in S has exactly one a.
   (b) No string in S has b and c next to each other.
   (c) Every string in S has the same number of b's and c's.
   (d) Every string in S has odd length.
   (e) None of the above are true.

5) Circle each item in the list below that is a required property of an algorithm.

(a) Data must be supplied to the algorithm.

(b) The algorithm must have a worst case scenario function that is $O(n)$.

(c) The algorithm must produce output.

(d) The algorithm cannot require infinitely many steps.

(e) The algorithm steps cannot be ambiguous.

6) The fact that for all integers a, b, c, it is true that $a(b + c) = ab + ac$ is called the

(a) Distributive Law

(b) Associative Law

(c) Commutative Law

(d) Inductive Law

(e) Identity Law

7) Which one of the following is <u>not</u> true about the divides relation: (all letters represent integers)

(a) $4|12 = 3$

(b) $0|0$

(c) For all integers a and b, $a|ab$.

(d) If $a|b$ and $a|c$ then $a|(b + c)$.

(e) For all integers a, $a| - a$.

8) If a, b, s, t are integers, and $as + bt = 4$, then (circle all that are true in the list below):

(a) $\gcd(a, b)$ could be 1

(b) $\gcd(a, b)$ could be 2

(c) $\gcd(a, b)$ could be 3

(d) $\gcd(a, b)$ could be 4

(e) $\gcd(a, b)$ could be 5

III. (10 points each) Problems

Do any three of the following four problems. If you do all four, I'll count your best three.

1) Use induction to prove that $1 + 2 + 3 + \cdots + n = \dfrac{n(n+1)}{2}$ for every positive integer n.

2) A sequence of integers is defined recursively by the rules $h_0 = 1$, and for $n \geq 1$, $h_n = 2h_{n-1} + 1$. Compute the h_1, h_2, h_3, h_4, h_5. Guess the simple closed form formula for h_n. Hint: It might help you guess the formula if you think about adding 1 to each of the terms h_0 through h_5.

3) Write an algorithm, as a sequence of steps, that will take two positive integers, m, n, as input, and produce n^m as output.

4) Compute $\gcd(1452, 531)$, and write the gcd as a linear combination of 1452 and 531.